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Fatigue impact from non-Gaussian random vibration

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Is it enough to define a Non-Gaussian vibration test signal by giving PSD and kurtosis?

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What is kurtosis?

Mathematically, the normalized fourth moment

$$kurtosis = \frac{mean(x^4)}{\sigma^4}$$

Tells how much high amplitude levels we have, compared to a Gaussian signal. Kurtosis for Gauss = 3, Mean is subtracted before calculation.

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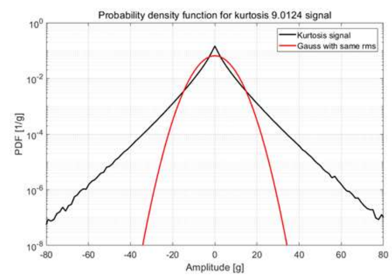
Example, kurtosis = 9.0124



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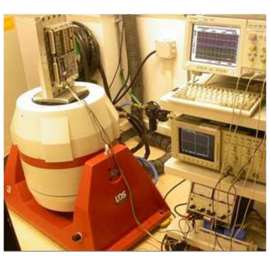

Will a high kurtosis test signal cause more damage than corresponding Gaussian? (same PSD)

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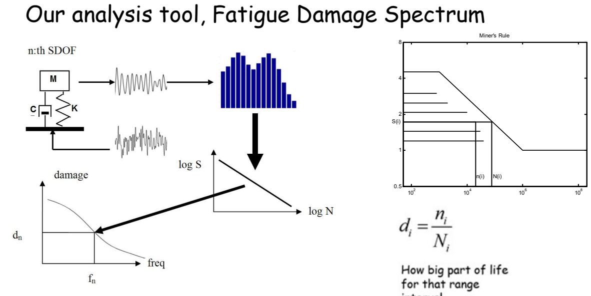
Our analysis tool, Fatigue Damage Spectrum



$$d_i = \frac{n_i}{N_i}$$

How big part of life for that range interval

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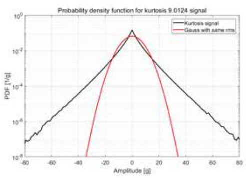
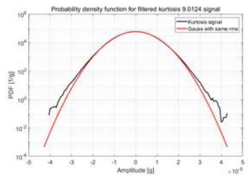
But hello! Doesn't filtering of a Non-Gaussian signal make it more Gaussian? Yes, (and no!). We filter our kurtosis nine signal in a relative displacement filter with f0 = 200 Hz, Q = 10.



Kurtosis 9.01



Kurtosis 3.13

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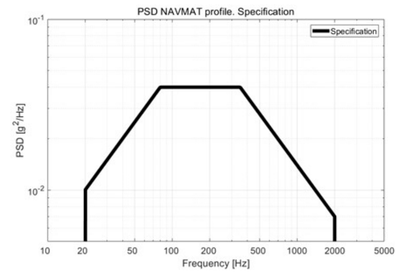
Hmmm! Let us see what we have done. We start with a test specification PSD, NAVMAT profile.



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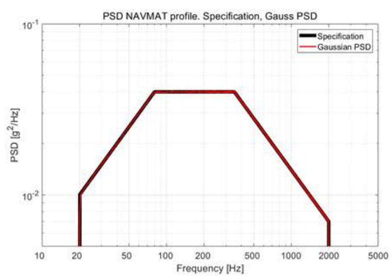
Make a Gaussian simulation with that PSD



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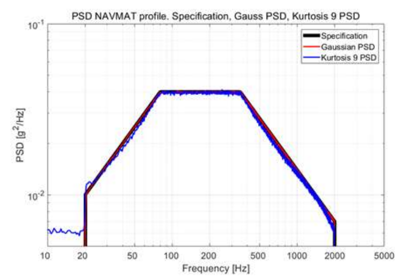
Make a high kurtosis simulation with kurtosis 9



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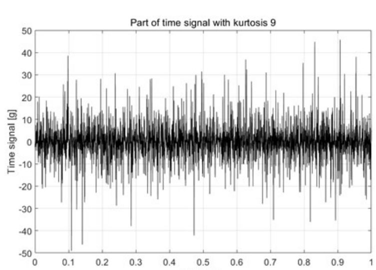
This was the signal becoming Gaussian when filtered!



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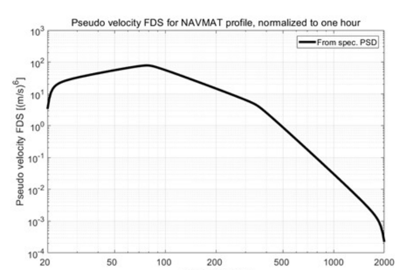
How about Fatigue damage Spectrum, FDS?.



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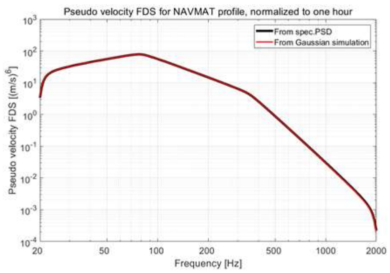
FDS for Gaussian simulation



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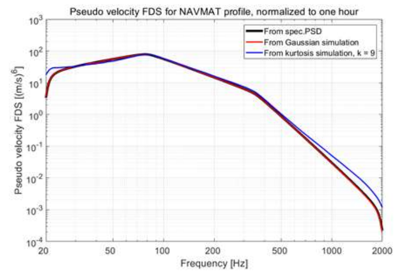
And for the high kurtosis signal



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The filtering of the signal makes it Gaussian, so no extra damage for high kurtosis! We don't have to bother about high kurtosis, *CASE CLOSED*! Time for coffee break!

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Wait a second!
There is more!

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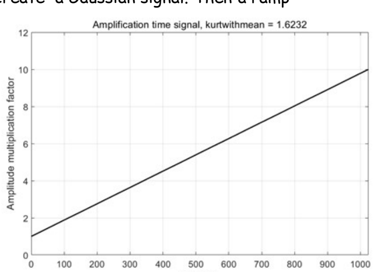
We look at a philosophical concept.
I used to show this to my students, when we discussed stationarity.
I create a Gaussian signal. Then a ramp



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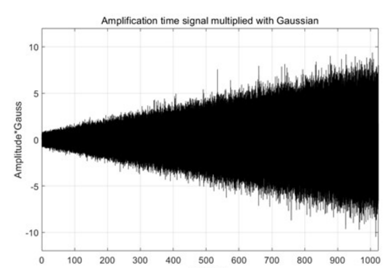
This is a simple example of a locally stationary signal with varying rms, studied by Martin O.



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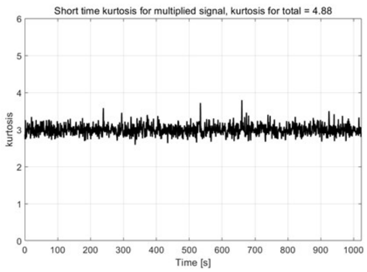
The multiplied signal is Gaussian all the time for short time intervals. But the kurtosis for the whole signal is 4.88



kurtwithm(w)*kurtosis(x)
4.8756

kurtosis(w.*x)
4.8836

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kurtosis with mean

```
function c = kurtwithm(x)
% KURTWITHM    kurtosis with mean
%          c = kurtwithm(x)
%          x  data vector
%             mean is not subtracted

c = sum(x.^4)*length(x)/sum(x.^2)/sum(x.^2);
```


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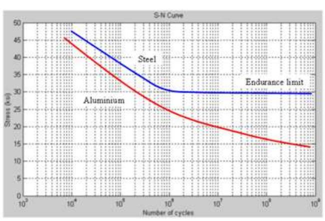
This also points to a major obstacle when it comes to testing and the use of FDS.
We can theoretically make a test signal with kurtosis 4.88 this way, BUT THE AMPLITUDE WILL VARY TEN TIMES, AND THE S/N CURVE USEFUL RANGE IS USUALLY MUCH SMALLER!



There are also other things to consider, such as crest factor, but here we concentrate on the math!

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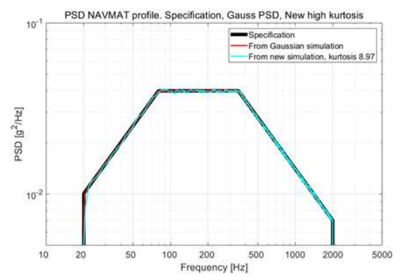
Let us make another simulation. PSD for new high kurtosis simulation looks OK.



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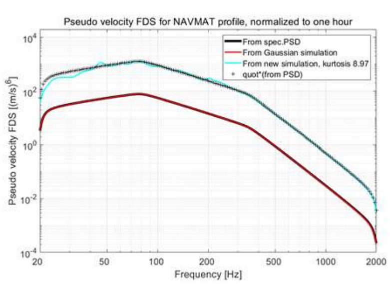
FDS for new high kurtosis simulation



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signal = (amplitude signal w) * (Gaussian signal g)
required kurtosis with mean for w is km = kurt/3

If the Wöhler exponent is b, then the FDS amplification referred to FDS for the Gaussian signal only is

$$quot = \frac{mean(w^b)}{(mean(w^2))^{\frac{b}{2}}}$$

The statistical properties of w determines quot, meaning kurtosis and b are not enough to determine quot!

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By using a slow amplitude function w with a Weibull distribution, we may calculate the quotient to FDS from Gaussian signal with same PSD.

```
function Qu = quot(kurt,b)
kk = kurt/3;
s = ['gamma(1+4/x)/(gamma(1+2/x))^2 - ' num2str(kk)];
k = fzero(s,1);
Qu = gamma(1+b/k)/(gamma(1+2/k))^(b/2);
```


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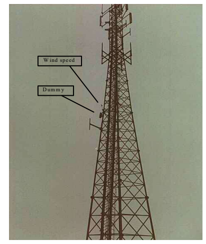
Some high kurtosis field examples

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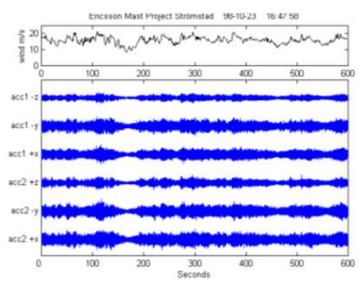

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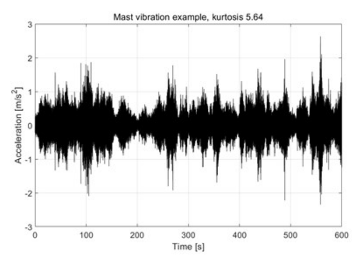
Idea: PSD is determined by structure, amplitude determined by wind



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MATLAB function *psdkurtsynt*

```
function [x,p,f] = psdkurtsynt(freq,vals,fs,N,kurt, ktol, wbcut,NN);
%PSDKURTSYNT Synthesizes a non-gaussian time signal from given psd and kurtosis
%         Compare psdsynt
%         A gaussian signal with given psd is made with psdsynt
%         A slowly varying amplitude signal with Weibull distribution
%         is multiplied onto the Gaussian signal to get specified kurtosis
%
%         [x,p,f] = psdkurtsynt(freq,vals,fs,N,kurt,ktol,wbcut,NN)
%         The spectrum is given as straight lines in log log
%         x       time signal vector
%         p       psd from freq, vals
%         f       frequency vector in Hz for p
%         freq    frequency corner points vector, first value must
%                 be near 0 and last value must be fs/2.
%         vals    psd value vector in corner points
%         fs      sampling frequency in Hz for x
%         N       size of x = 2^N
%         kurt    specified kurtosis
%         ktol    kurtosis tolerance in %
%         wbcut   cutoff frequency in Hz for multiplying Weibull signal   <----
%         NN      downsampling factor to create Weibull signal
```


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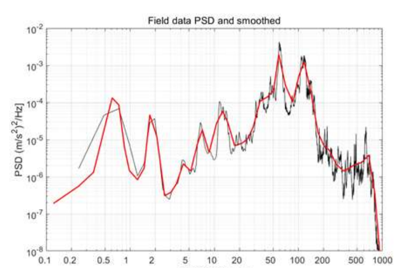
Look at the field data example



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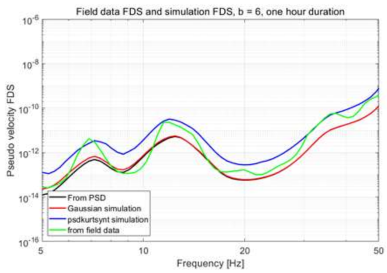

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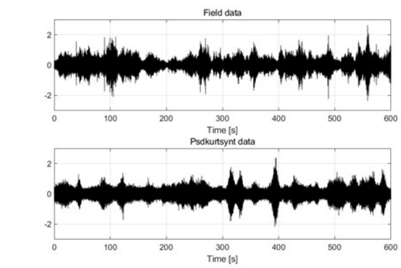

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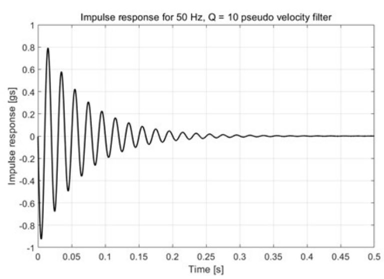
What is the running rms for our two signals with kurtosis nine?



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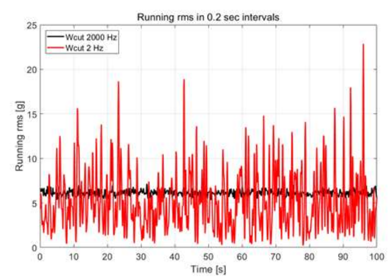
Running rms in 0.2 sec intervals



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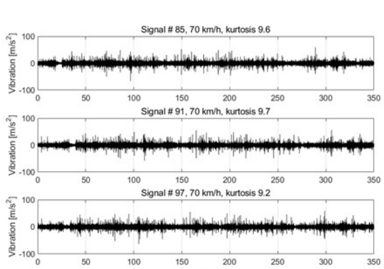
Spiky field measured signals. Measured on truck frame while driving on bad road. Pick out three recordings with speed 70 km/h



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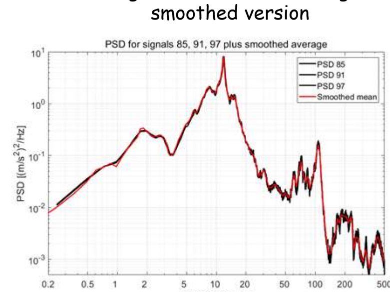
PSD for the signals are calculated, together with a smoothed version



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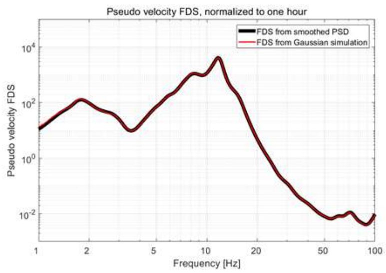
Pseudo velocity FDS, from PSD and from Gaussian simulation with given PSD



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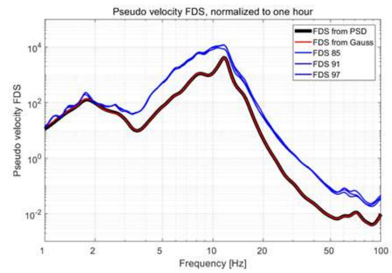
ADD FDS from field data



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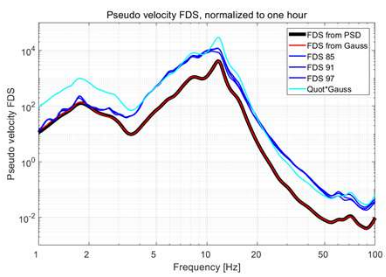
If signal was made with multiplication method, Quot would be 7.2



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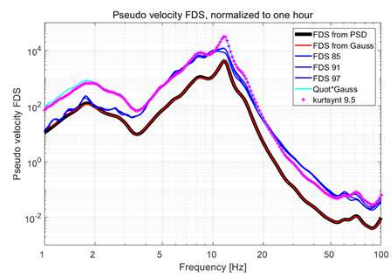
Add *kurtsynt* 9.5 with wcut = 0.1 Hz



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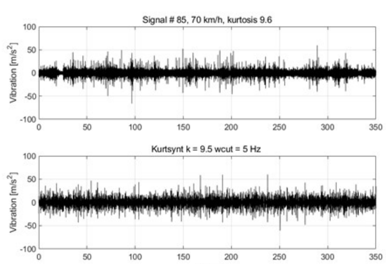
With wcut = 5 Hz, it looks quite like the field signal



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Fatigue Impact from Non-Gaussian Random Vibration

or

Is it enough to define a Non-Gaussian vibration test signal by giving PSD and kurtosis?

NO, some extra information needed!

FDS???

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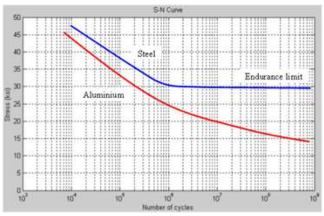
Back to the limited range of the straight line Wöhler approximation.
There are suggestions, such as Haibach. But how to determine the knee?



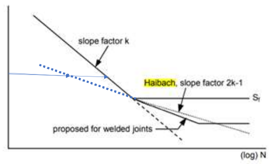


Fig.13: S-N curve extrapolated below the fatigue limit.

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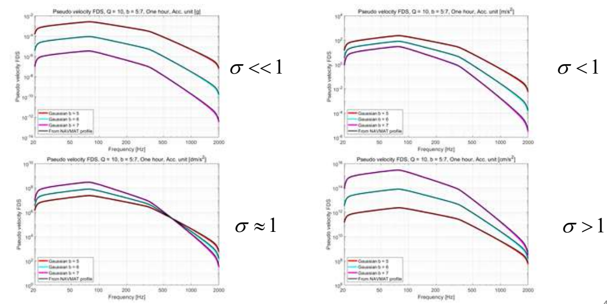
Which brings us to the scaling, or units. NAVMAT PSD, Gaussian simulation



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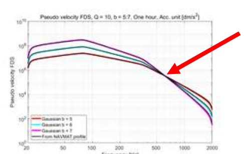
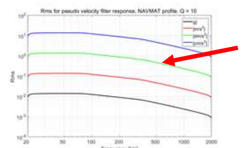
From psd2fds: *s(n,1) = T*sqrt(m2/m0)*(m0*2)^(b/2)*gamma(b/2+1)*
m0 = (rms)^2 What rms for same FDS for b = 5 and b = 7?

$$(2\cdot\sigma^2)^{\frac{5}{2}}\cdot\Gamma(5/2+1) = (2\cdot\sigma^2)^{\frac{7}{2}}\cdot\Gamma(7/2+1)$$

$$2\cdot\sigma^2 = \frac{\Gamma(5/2+1)}{\Gamma(7/2+1)} = 0.2857;\quad \sigma = 0.38$$





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Some people use a stiffness factor K. Common value is 1000.
Why not use SI unit [mm] for relative displacement and [mm/s] for pseudo velocity ?

Anyway, there is a great need for a standard for FDS!

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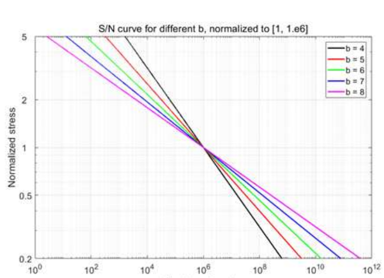

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