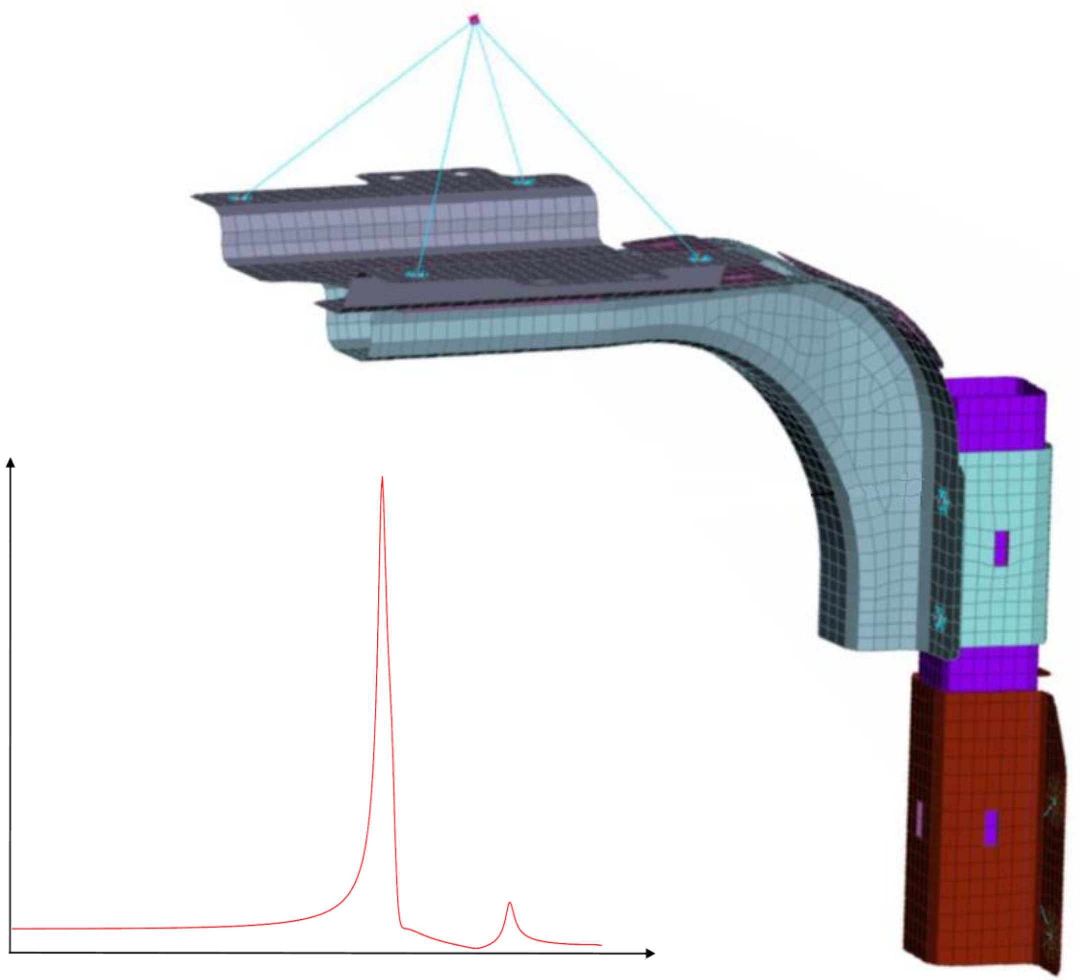
Virtual random vibration testing with FEM

Why do we need it and how do we do it?



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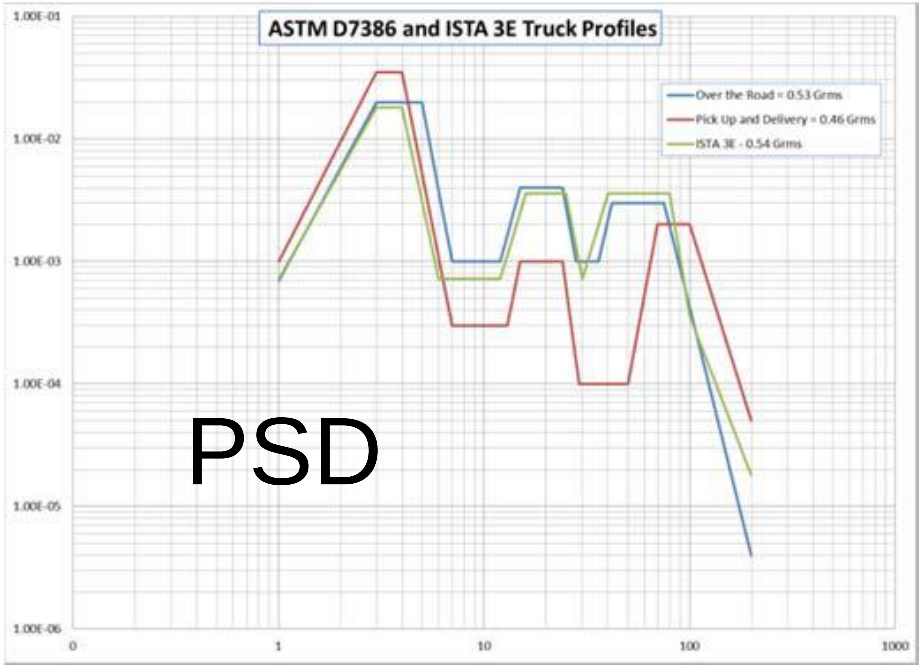
Fatigue damage estimate from random vibration

- Stationary Gaussian vibration → Power spectral density (PSD)

$$m_k = \int_{-\infty}^{\infty} G_{xx}(f)\,|f|^k \,\mathrm{d}f$$

- Extract statistical properties of the vibration using moment of area
- Expected fatigue damage per unit time of scalar (uniaxial) stress PSD, using a spectral method (Rayleigh approximation, Dirlik, etc.)
- Commercial tools for vibration fatigue analysis have been available for a long time



Vibration duration

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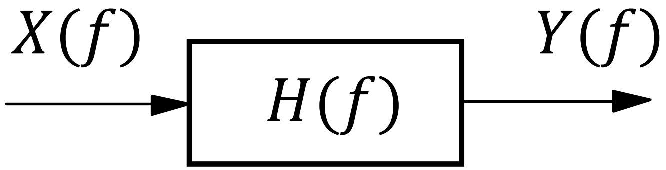
$\mathrm{E}[D]$

Motivation for frequency domain analysis

- Commercial random vibration fatigue tools are promoted with benefit in computation time
- No need for transient FEA, that is time demanding for large models and long vibration durations
- Instead, Frequency Response Functions (FRF = transfer functions, $H(f)$) are quickly calculated and used for equally quick stress response calculation in the frequency domain.



The Fourier transform of the stress output $Y(f)$ equals the product of the Fourier transform of the excitation, $X(f)$, and the FRF, $H(f)$

In the case of Gaussian stationary random vibration,

$$Y(f) = H(f) \cdot X(f)$$

$$G_{YY}(f) = H(f) \cdot G_{XX}(f) \cdot H(f)^{\dagger}$$

Linear system assumed

SIMPLE AND FAST

Applicability

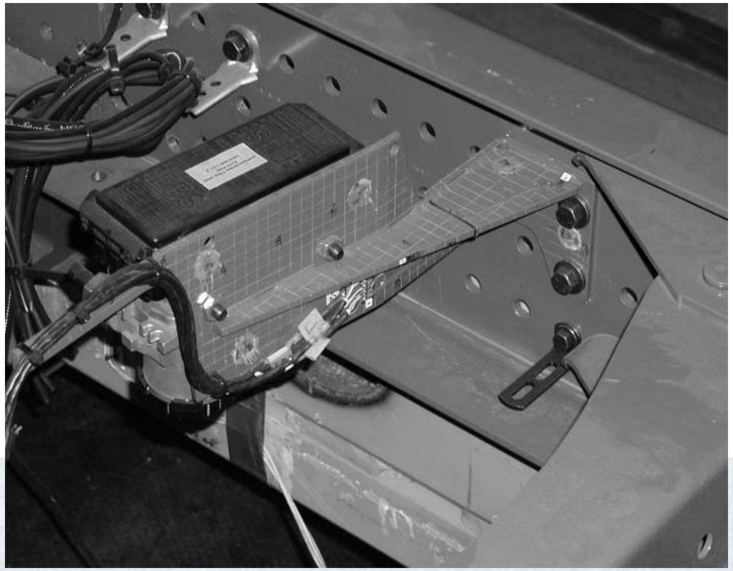
- Industrial use of frequency domain methods is limited! (or is it?)
  - Engineers prefer deterministic analysis
  - Modern computers are fast (transient FEA more feasible)
  - Loading is not often a true stationary random vibration

- It should be used when you do have true stationary random vibration

- … but be careful when you have a vibration that is not really stationary random!
  - Do not calculate PSD average!
  - It is possible to derive a PSD (for a stationary random vibration) that is damage equivalent with any type of vibration input, through comparison of Fatigue Damage Spectrum (FDS)

Applicability for a truck company

- Imagine the development of a truck with numerous components mounted to the chassis or cab structure
- A component fatigue requirement is simplified using reference to a complete truck test method on a durability test track
  - Physical measurement of vibration input
  - Truck simulation on virtual test track

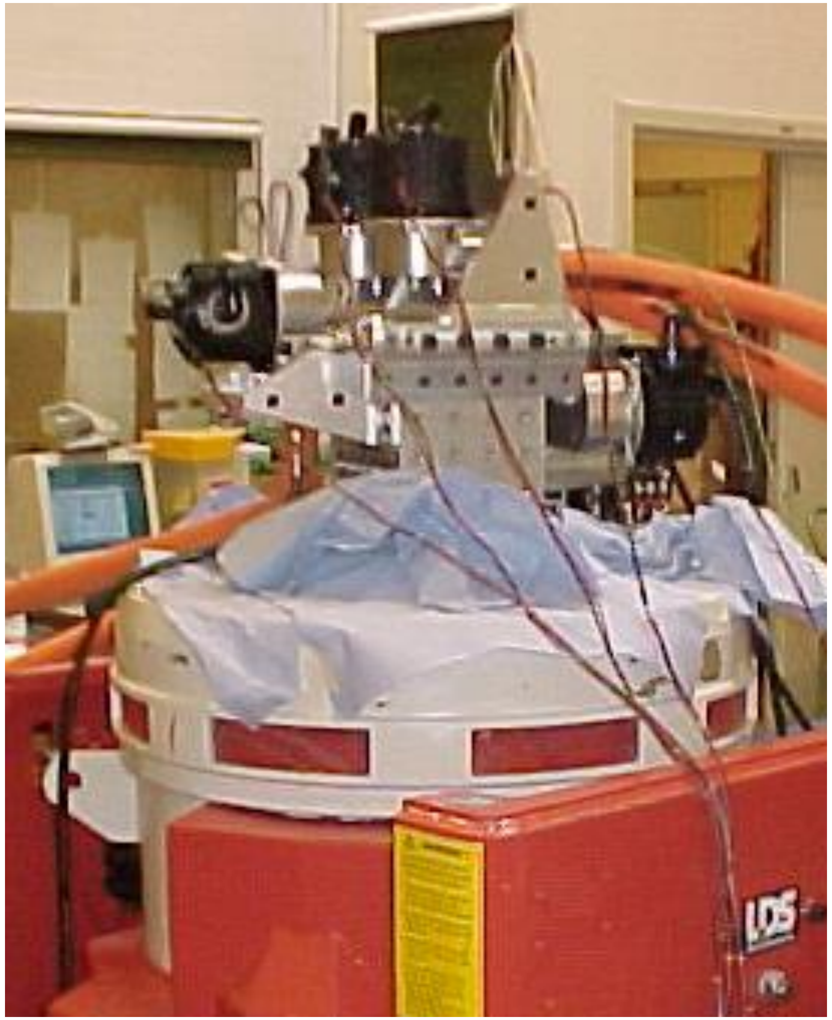


Applicability for a truck company

- ...but what if an external supplier is responsible for development of the component?
  - It is difficult for the supplier to assess the fatigue requirement with the test track as reference
  - The track gives a complex loading description
  - The test track obstacles are not something the truck company want to share information about
- Further load simplification is needed for early prototyping by the supplier!

Applicability for a truck company

- A proposal is to use a fatigue requirement with stationary random load (PSD)
- As is already done for physical testing on vibrators, so why not?
- Easy to understand and well-defined requirement
- The original requirement has mostly random-like vibration
- Non-stationarity can be taken into account through damage-equivalence, with reference to FDS
- … but it needs to be conservative because of the simplification

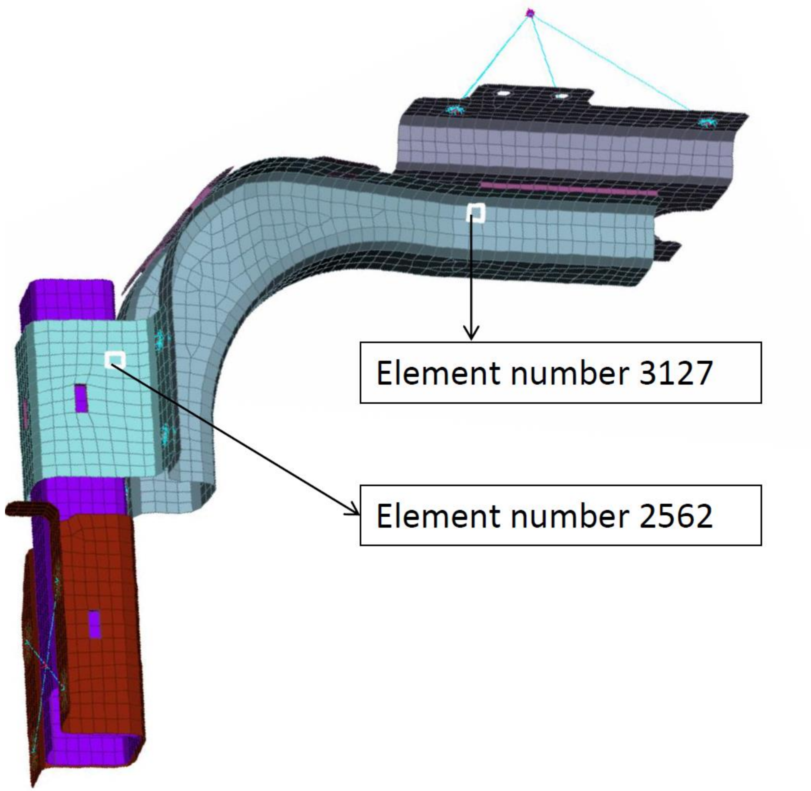


Virtual random vibration testing

- If having acquired PSD and linear FE-model of the component, why not perform a virtual vibration testing also?
- The same PSD profile used for vibrators can be used as input excitation in a linear FE-model
- Perform a modal analysis and select all eigenmodes that are excited by the input PSD
- Superimpose the modal transfer functions to acquire the transfer function matrix $H(f)$ between input $X(f)$ and stresses $Y(f)$ at selected points
- Multiply the input PSD with the transfer functions $H(f)$ to get the stress PSD matrices:

$$G_{YY}(f) = H(f) \cdot G_{XX}(f) \cdot H(f)^{\dagger}$$

$$\mathbf{M}\,\ddot{\mathbf{q}} + \mathbf{C}\,\dot{\mathbf{q}} + \mathbf{K}\,\mathbf{q} = \mathbf{f}$$



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Multiaxial stress

- A challenge concerns the general case when the critical stress response is not necessarily a uniaxial, scalar stress.
- Often there is one principal stress that dominates the stress response. However, when that is not the case the method must be able to handle complex stress states, including multiaxial stresses with rotating principal directions.

$$X(f) \longrightarrow \boxed{H(f)} \longrightarrow Y(f) = \begin{bmatrix} \sigma_{11}(f) \\ \sigma_{22}(f) \\ \sigma_{12}(f) \end{bmatrix} \qquad G_{YY}(f) = H(f) \cdot G_{XX}(f) \cdot H(f)^{\dagger}$$

3×3 PSD matrix

Multiaxial stress

- A stress response is determined by a combination of modal stresses
- Even for one excitation (direction), we can have multiaxial stress response
- The multiaxial stress response can even be non-proportional when modal stresses have different stress directions for the major principal stress
- Transfer functions derived using FEA will give you the general stress response in terms of a matrix of auto- and cross-PSD of normal and shear stresses
- …so, how do we quantify and compare the fatigue impact from different PSD matrices representing the ’response’ ?

Reduction to a scalar stress measure

- Fatigue impact simplified to 'standard' uniaxial fatigue calculation
- Alternative 1: projected stress in the direction with max RMS
  - iterative procedure to find direction (or same as 1st principal stress for dominating mode(s))
  - multiaxiality ignored
- Alternative 2: <u>Equivalent</u> von Mises stress
  - Pitoset & Preumont, Int. J of Fatigue (2000)
  - captures multiaxiality while retaining frequency distribution

Equivalent von Mises stress

- von Mises (biaxial):

$$\sigma_e^2 = \sigma_{11}^2 + \sigma_{22}^2 - \sigma_{11}\sigma_{22} + 3\sigma_{12}^2$$
$$= y^T Q y = Q : (yy^T)$$

$$y = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix}, \qquad Q = \begin{bmatrix} 1 & -1/2 & 0 \\ -1/2 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

- Consider its mean square of the von Mises stress. Since $E[\cdot]$ is a linear operator, we have that

$$E[\sigma_e^2] = Q : E[yy^T]$$

where $E[yy^T]$ is the covariance matrix, related to the stress PSD matrix $G_{YY}(f)$ through the integral

$$E[yy^T] = \int_{-\infty}^{\infty} G_{YY}(f)\, df$$

Equivalent von Mises stress

- Now, let us define an equivalent (scalar) PSD $G_{eq}(f)$ with mean square equal to $E[\sigma_e^2]$

$$E[\sigma_e^2] = \int_{-\infty}^{\infty} Q : G_{YY}(f)\, df = \int_{-\infty}^{\infty} G_{eq}(f)\, df$$

- Assuming equal integrands, the equivalent von Mises stress is defined as

$$\boxed{G_{eq}(f) = Q : G_{YY}(f)}$$

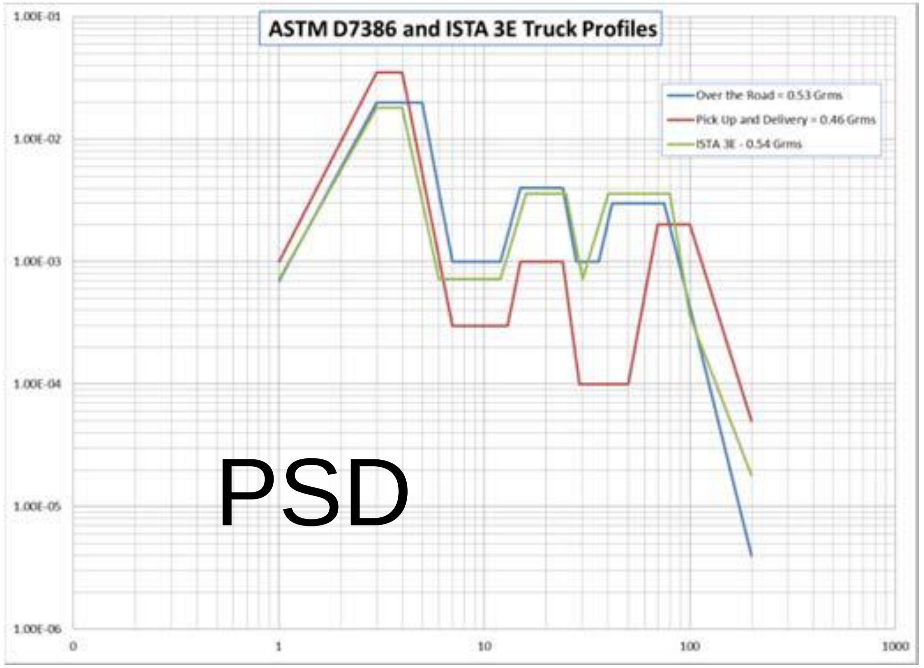
- … or expressed with a trace operator

$$\boxed{G_{eq}(f) = \mathrm{trace}\{Q \cdot G_{YY}(f)\}}$$

- Same mean square as the von Mises stress
- It reduces itself to the uniaxial alternating stress if the stress response is uniaxial

Problem solved – continue with fatigue result

- …and we are back to the classical uniaxial vibration fatigue problem
- …that relate a scalar stress PSD to expectation of fatigue damage per time unit



Vibration duration

$$\mathrm{E}[D]$$

Thank you!