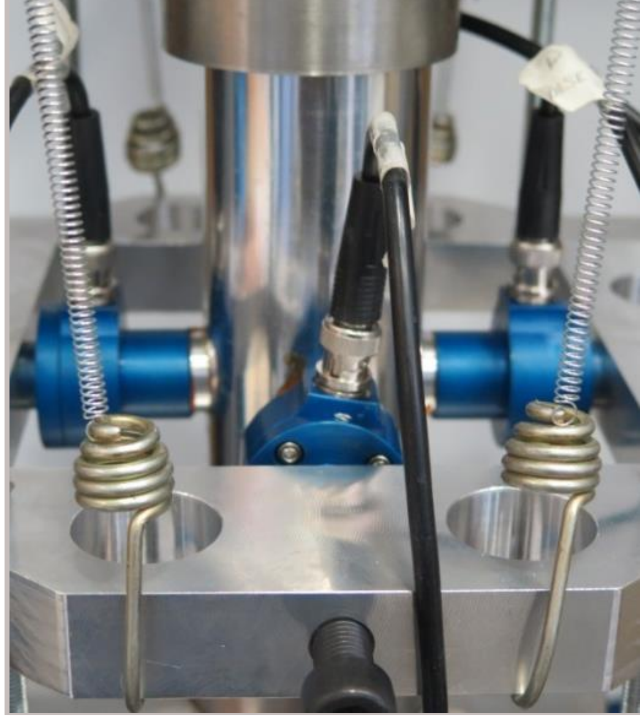
Non-stationary field data and response spectrum analysis



RISE Research Institutes of Sweden

Martin Olofsson

Chemistry & Applied Mechanics

RISE Research Institutes of Sweden

- State-owned research institute with a mission to be a strong innovation partner to corporations and society
- 2700 employees offer unique expertise in a wide range of knowledge and application fields (1/3 with a PhD)
- 100 testbeds and demonstration facilities

Short facts about RISE Applied Mechanics

- 50 researchers, engineers, technicians and admin staff
- Node for solid and structural mechanics inside RISE
- Large experimental & simulation capabilities
- Expertise in shock & vibration integrity and reliability

Example of load frame for material testing

Max force 1.2 MN

Simulation of non-stationary random excitation

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Presented at 'Vibrationsdagen' November 11, 1996, arranged by SVIB

Abstract

This paper reviews a simulation procedure to use when developing laboratory fatigue tests, for example tests on an electrodynamic shaker. More specifically, it describes the specification of a shaker drive signal, or a control transducer signal, when a non-stationary stochastic excitation is to be reproduced. Road excitation of a vehicle is a typical excitation of this kind. The resulting simulation signal is made of a stationary Gaussian random realization multiplied with an amplitude modulating function (or 'running RMS').

The study was done at Volvo Car Corporation as a part of a Masters degree thesis project, for Chalmers University of Technology, Sweden. Together with co-author Thorbjörn Lundqvist the thesis 'Livslängdsprovning på vibrator av vibrationsutsatta komponenter' was published in April 1995 (in Swedish).

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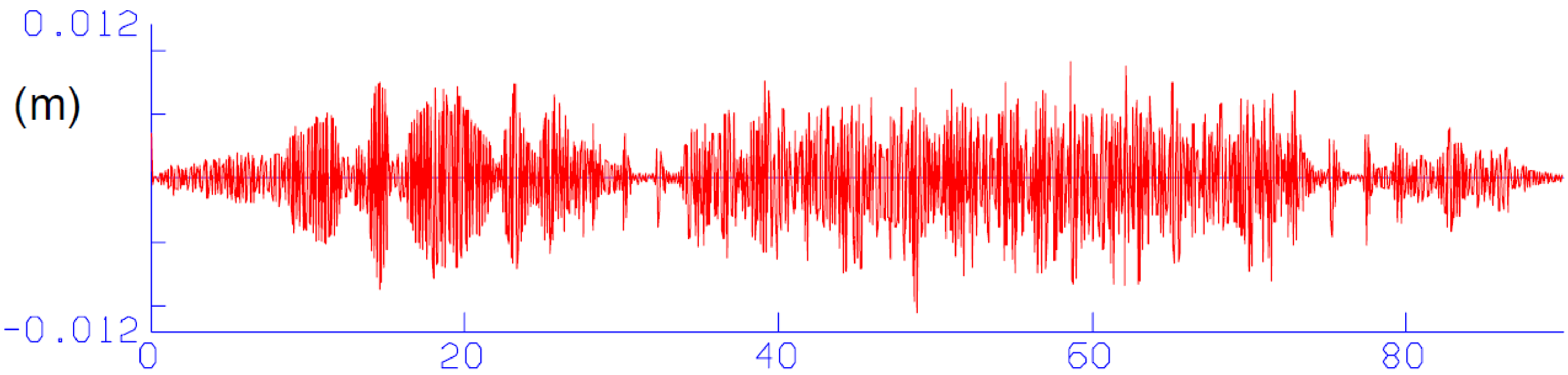
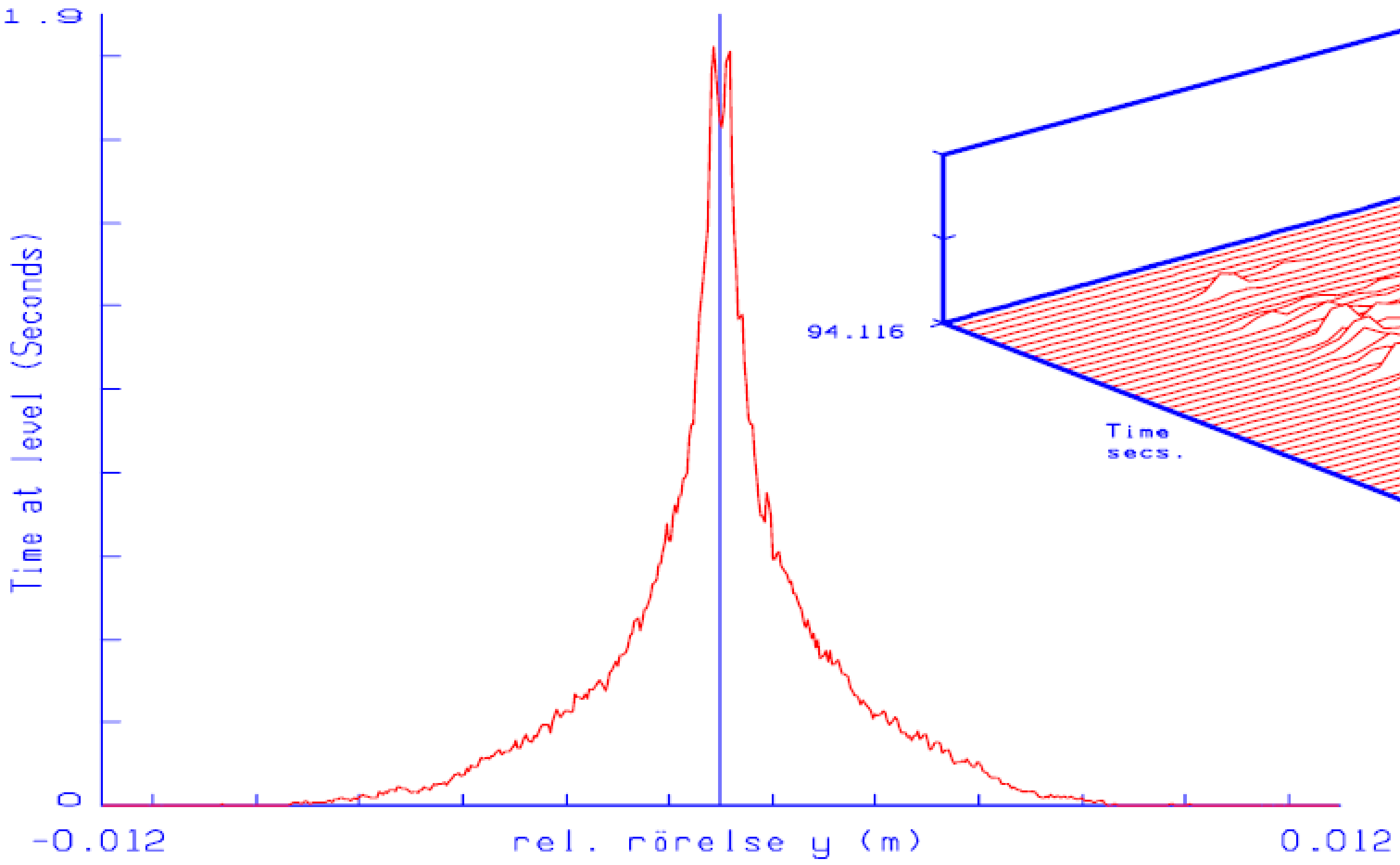
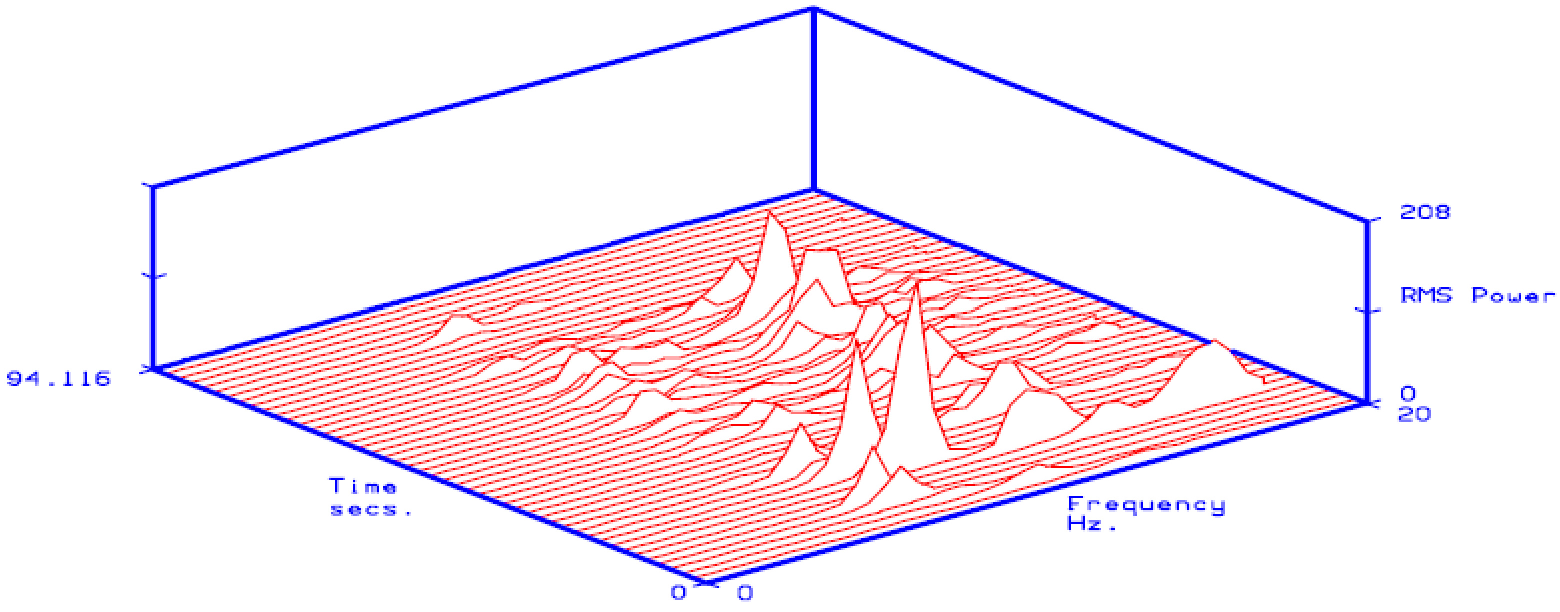
Typical measurement from a durability life target

- In order to capture enough data that can represent a durability life target, e.g. data from a durability test track, several different type of field events are measured
- Events can be stationary random, in parts, but can also include periodic vibration and sudden shocks
- It is clear that when you treat it as a stationary random vibration and process data into a PSD average, important information about damaging potential is lost



Not stationary – what about Gauss?





It is natural to be Gaussian

- especially if you are a complex individual composed as result from different sources

- Central Limit Theorem:
  - When independent random variables are summed up, their sum tends toward a normal distribution (informally a bell curve) even if the original variables themselves are not normally distributed

  https://en.wikipedia.org/wiki/Central_limit_theorem

- If you think you are dealing with a non-Gaussian vibration, the first question to ask yourself is if the vibration really is a stationary random vibration

The mathematical data model

- The 'product model' describes a zero mean non-stationary random process *X(t)*, as a product of a stationary random process *U(t)* and a deterministic non-negative function *a(t)*

$$X(t) = a(t)U(t)$$

- Without loss in generality, *U(t)* may be chosen to have unit variance. Thus, *a(t)* can be interpreted as the time-varying RMS value of the non-stationary random process *X(t)*

$$\mathrm{E}[U(t)] = 0,\ \mathrm{E}[U^2(t)] = 1 \qquad \Rightarrow \qquad \mathrm{E}[X(t)] = 0,\ \mathrm{E}[X^2(t)] = a^2(t)$$

Locally stationary process

- The Bendat and Piersol bible ‘Random Data’ states that if *a(t)* is varying with a frequency much lower than the frequency of *U(t)*, the autocorrelation of a ‘locally stationary’ random process *X(t)* can be approximated as

$$R_{XX}(\tau, t) \approx a^2(t) R_{UU}(\tau)$$

- The evolutionary power spectral density can be written as

$$S_{XX}(f,t) = \int_{-\infty}^{\infty} R_{XX}(\tau,t) e^{-i2\pi f\tau} d\tau \approx a^2(t) \int_{-\infty}^{\infty} R_{UU}(\tau) e^{-i2\pi f\tau} d\tau = a^2(t) S_{UU}(f)$$

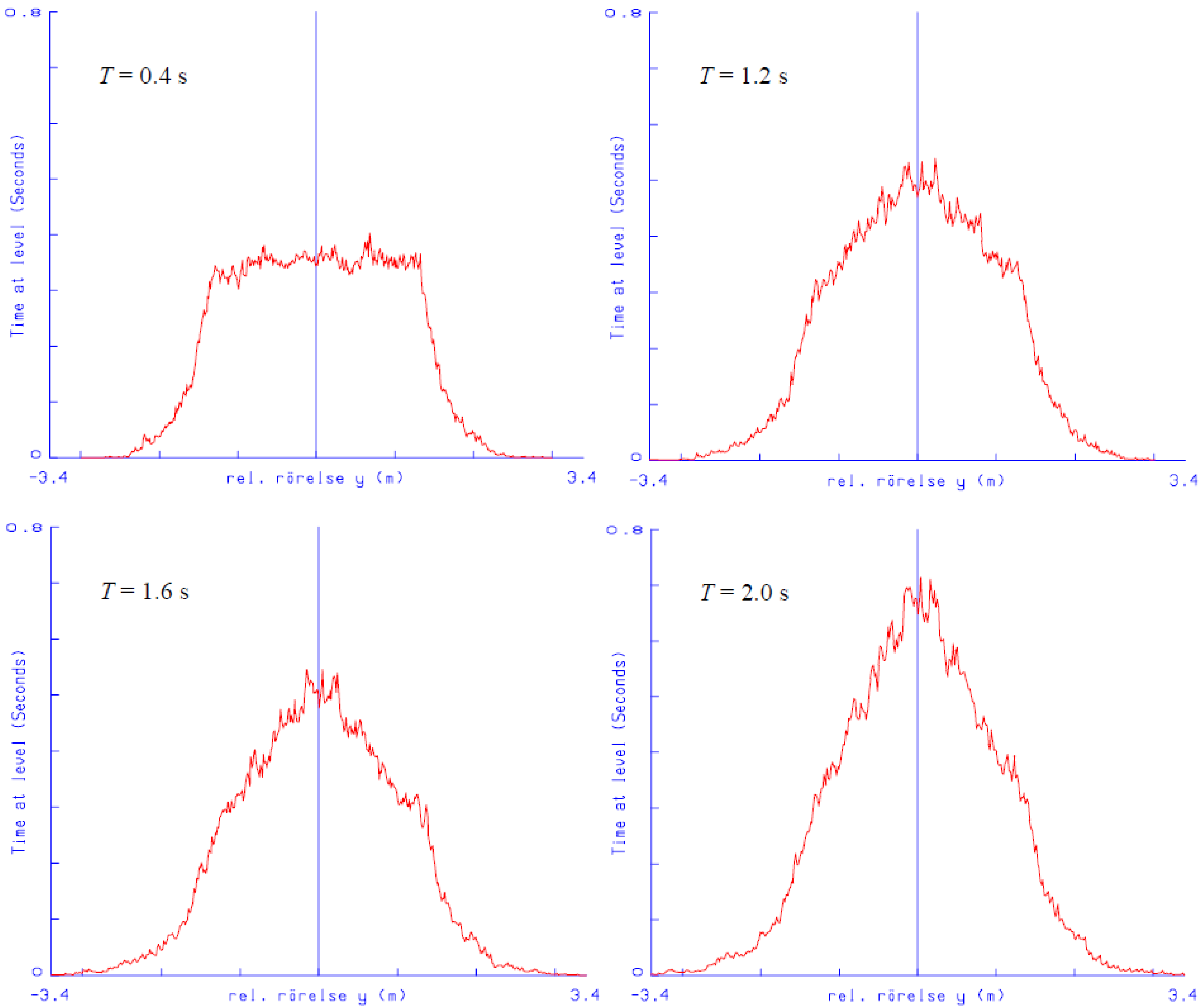
Estimation of running RMS, $a^*(t)$

- The RMS value of all samples in a time interval T was calculated, as the interval was moved along $x_m(t)$

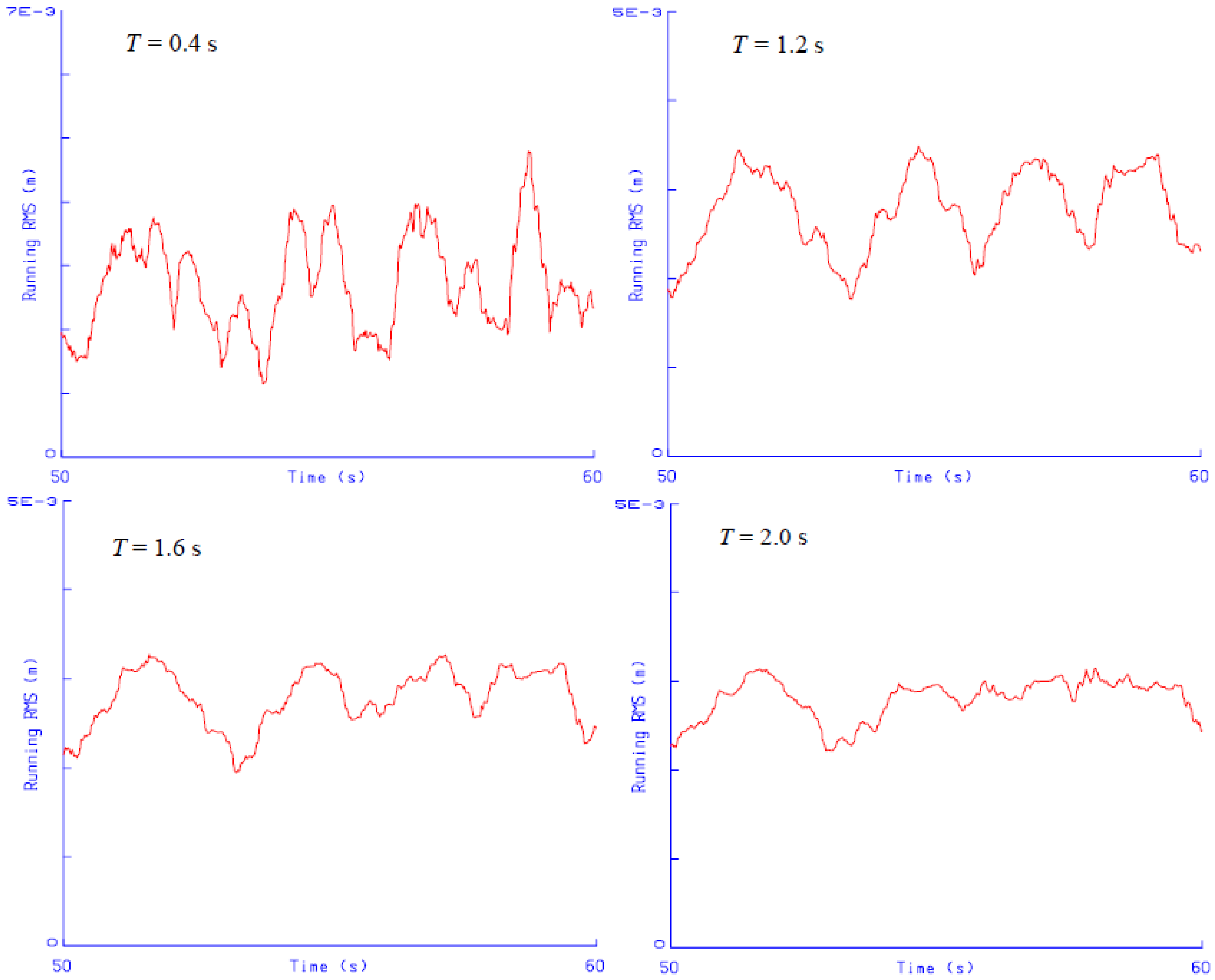
$$a^*(t) = \sqrt{\frac{1}{T}\int_{t-T/2}^{t+T/2} x_m^2(\tau)\,d\tau}$$

- Now, let's assume the vibration is 'natural', i.e. $U(t)$ is Gaussian! Then, let's try different time-intervals for RMS calculation and have a look at the time-at-level histogram for

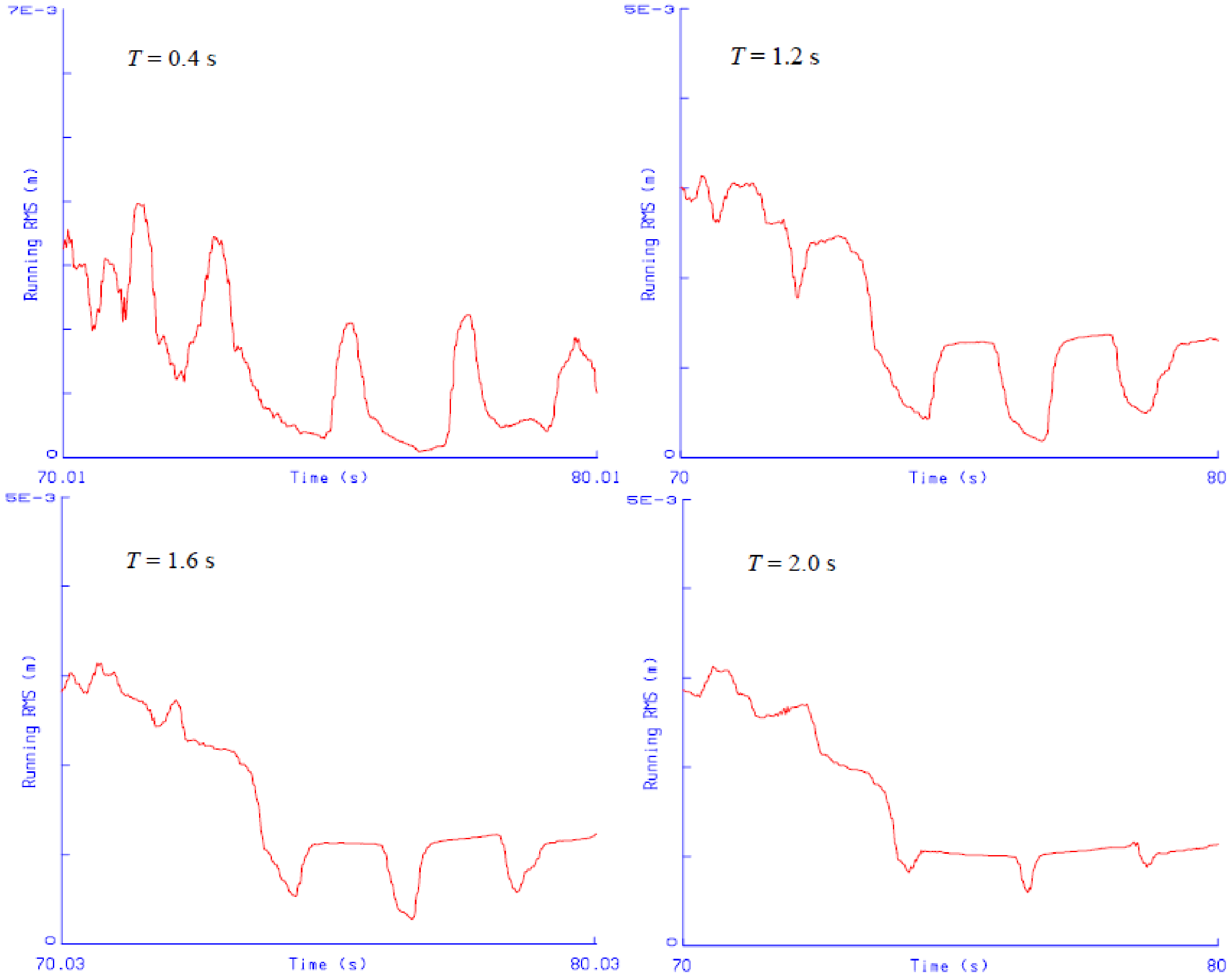
$$u(t) = x_m(t) \,/\, a^*(t)$$


T = 0.4 s
T = 1.2 s
T = 1.6 s
T = 2.0 s
Time at level (Seconds)
rel. rörelse y (m)
0.8
0
-3.4
3.4

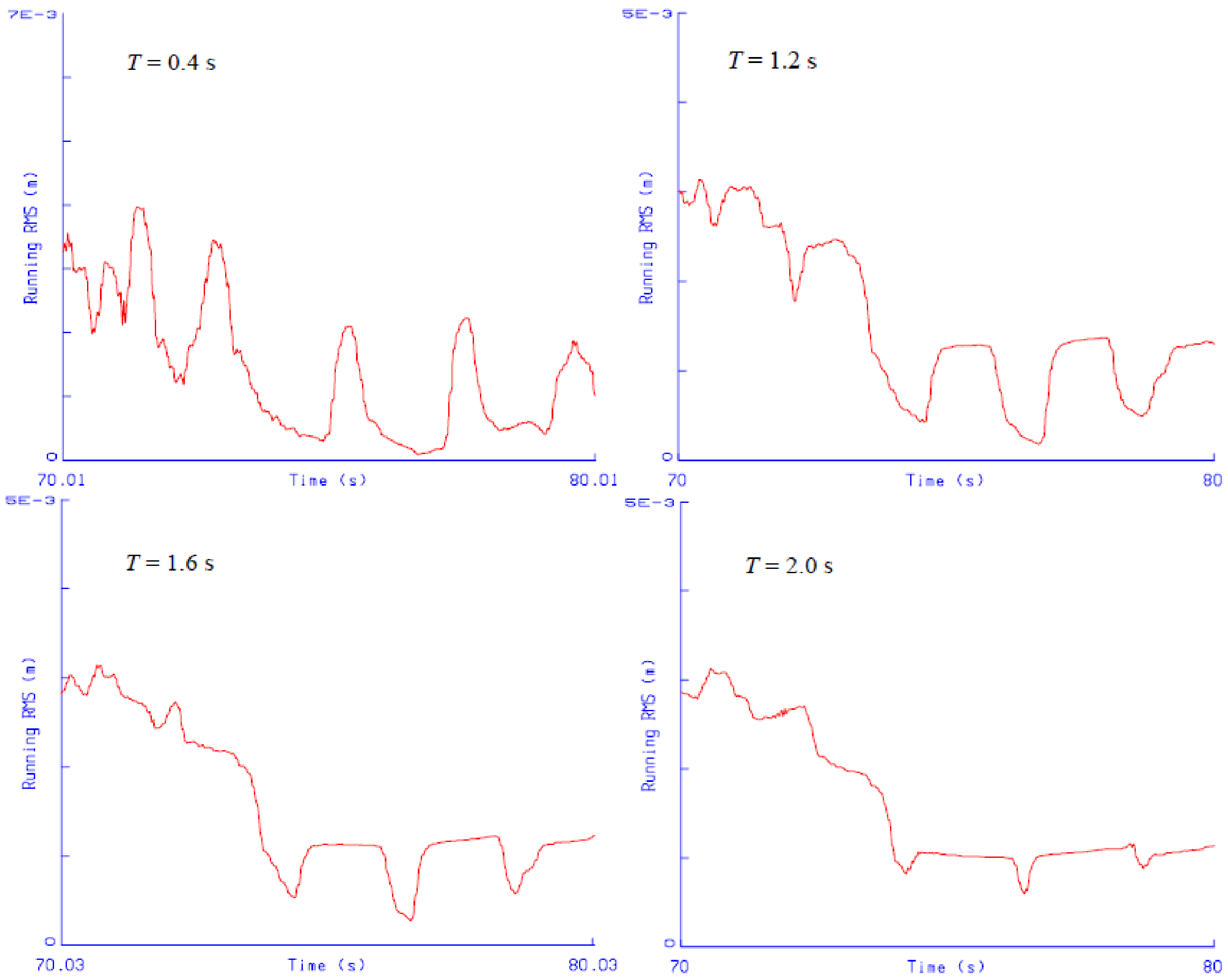
$a^*(t)$ from ’almost stationary’ test track data



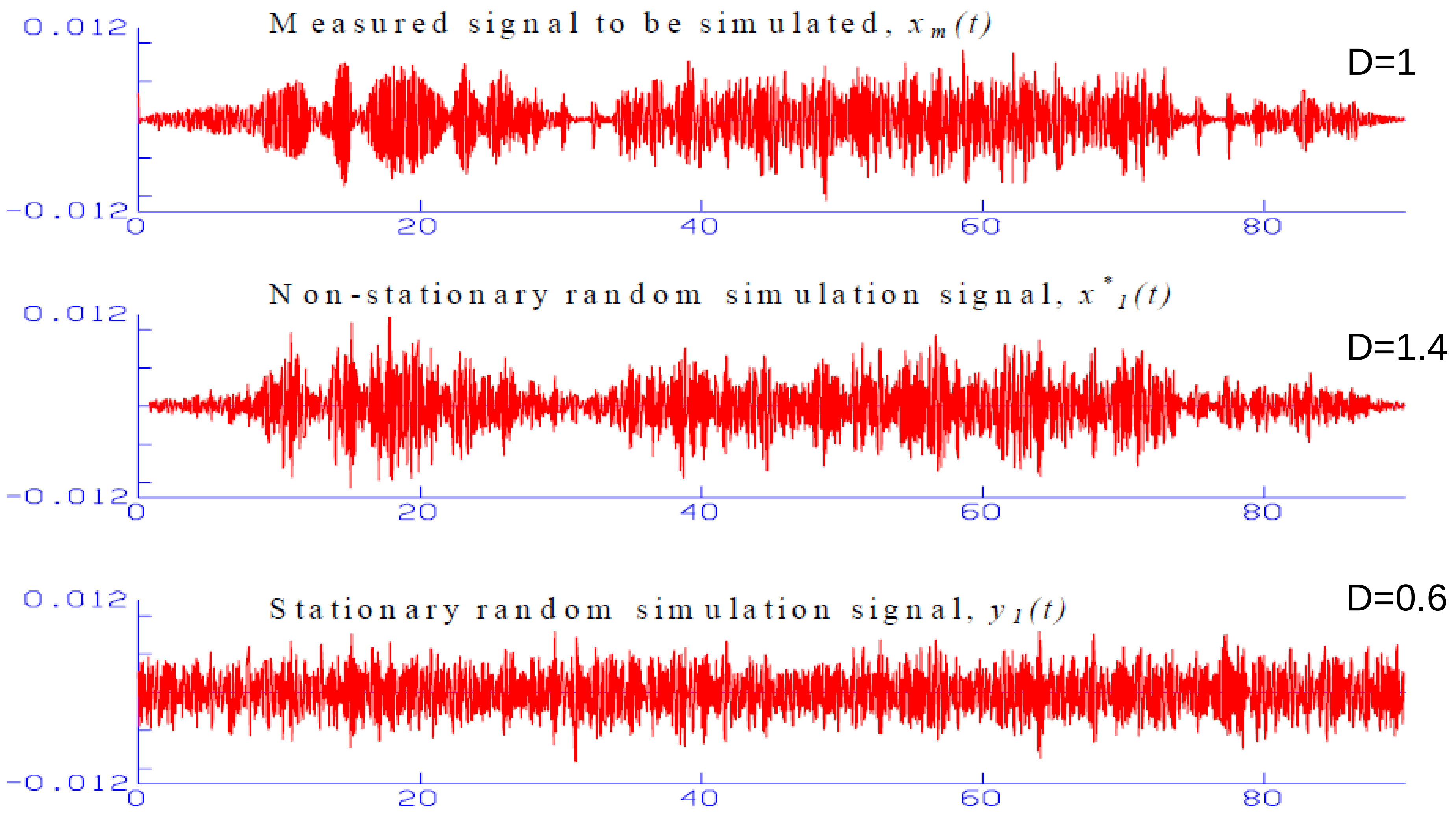
$a^*(t)$ from test track data with transient 'potholes'



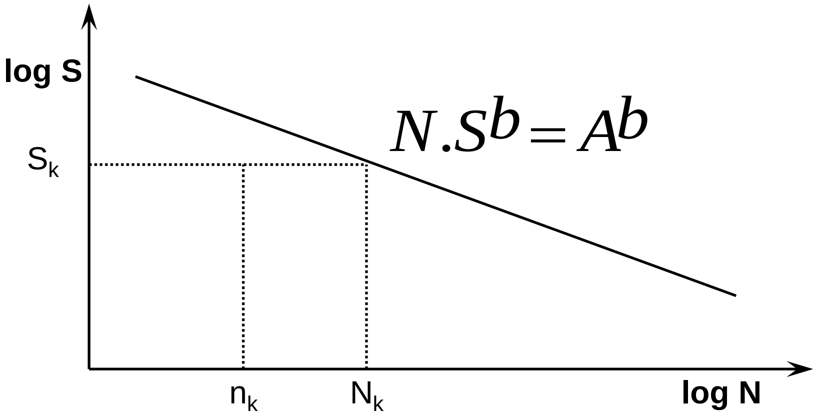
$a^*(t)$ from test track data with transient 'potholes'



Simulation result product model and T=1.6 s



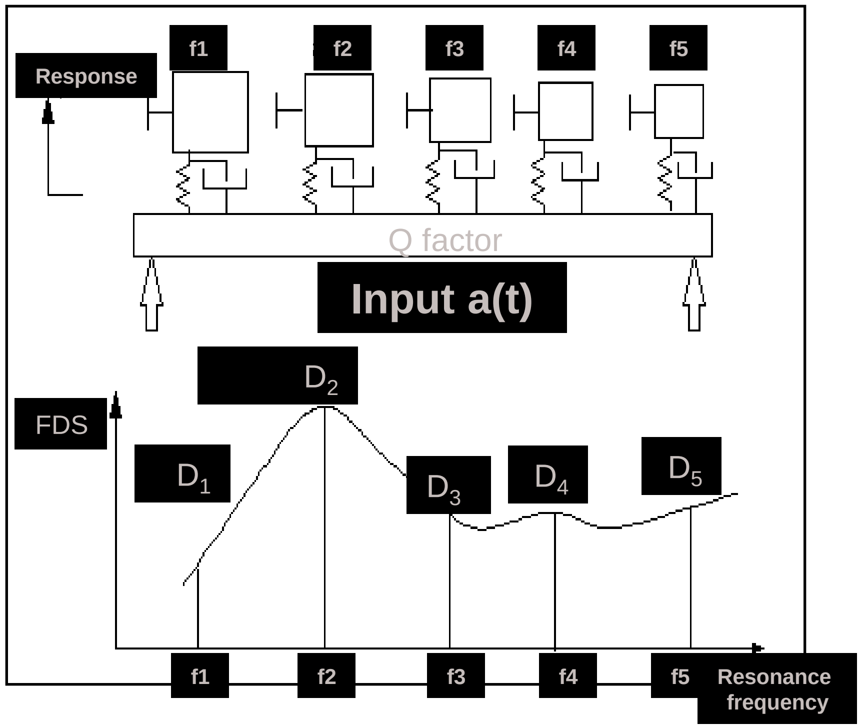
Fatigue Damage Spectrum, FDS



Wöhler curve

$$D_i = \sum_k \frac{n_k}{N_k}$$

Miners rule



Damage-equivalent stationary vibration through FDS

- Alternative to nonstationary simulation is to find a stationary one that impose the same fatigue damage of the component, regardless of what (resonance) frequency the component is sensitive to.

1. Calcutate FDS from the vibration input
2. Back-calculate PSD for a stationary random vibration, from the FDS

Thank you!

- Questions are welcome!